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General $\mathcal{N} = 1$ Supersymmetric Fluxes in Massive Type IIA String Theory

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ABSTRACT

We study conditions on general fluxes of massive Type IIA supergravity that lead to four-dimensional backgrounds with $\mathcal{N} = 1$ supersymmetry. We derive these conditions in the case of SU(3)- as well as SU(2)-structures. SU(3)-structures imply that the internal space is constrained to be a nearly Kähler manifold with all the turned on fluxes, and the negative cosmological constant proportional to the mass parameter, and the dilaton fixed by the quantized ratio of the three-form and four-form fluxes. We further discuss the implications of such flux vacua with added intersecting D6-branes, leading to the chiral non-Abelian gauge sectors (without orientifold projections). Examples that break SU(3)-structures to SU(2)-ones allow for the internal space conformally flat (up to orbifold and orientifold projections), for which we give an explicit example. These results provide a starting point for further study of the four-dimensional (chiral) $\mathcal{N} = 1$ supersymmetric solutions of massive Type IIA supergravity with D-branes and fluxes, compactified on orientifolds.

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1 Introduction

Insights into four-dimensional $\mathcal{N} = 1$ supersymmetric vacua of M- and string theory with non-Abelian gauge sectors and chiral matter may provide an important link between the M-theory and the particle physics, describing the Standard Model and/or Grand Unified models. Within M-theory unification the perturbative heterotic string is only one of the “corners” of M-theory. Other corners, such as Type I, Type IIA and Type IIB superstring theory provide other, potentially phenomenologically viable string vacua, which are related to the heterotic ones via a web of string dualities. In the latter framework D-branes play an important role in constructing chiral models with non-Abelian symmetry.

The rich structure of $\mathcal{N} = 1$ supersymmetric vacua can be significantly increased by introducing, in addition to brane configurations, the (supergravity) fluxes. [There is a growing literature on the subject of string compactifications with fluxes which was initiated in [1, 2]. A partial list of subsequent works includes, e.g., [3]–[9], and for recent work, quantifying effects in terms of deformations of the original manifold (G-structures)[10], see [11]–[27] and references therein.] Typically fluxes generate a back reaction on the original geometry of the internal space, thus changing the nature of the internal space. The existence of Killing spinors in turn allows for the classification of the new geometry of the internal space in terms of specific non-trivial torsion components, which can be classified with respect to the structure group of the internal space (the so-called G-structures). [For a review see [16] and references therein.]

Another important effect of supergravity fluxes is the lift of the continuous moduli space of the string vacua, i.e, fluxes introduce the supergravity potential for the compactification moduli fields in the effective four-dimensional theory. The ground state solution, at the minimum can in principle preserve supersymmetry and at the same time fix a (sub-)set of compactification moduli. [For the intriguing new developments in the study of flux vacua with broken supersymmetry, see [28, 29, 30, 31] and references therein.] Thus, on one hand, the flux compactifications provide a mechanism for moduli stabilization, one essential ingredient in the construction of phenomenologically viable string vacua, and on the other hand, the (probe) D-branes sectors

provide the non-Abelian gauge structure and chiral matter, another essential ingredient in reproducing the realistic particle physics from M/string theory. Therefore the ultimate goal of the program is to obtain consistent, explicit constructions of (supersymmetric) flux vacua with D-brane configurations, whose fluxes would stabilize (most/all) moduli and the D-branes would reproduce the gauge structure and chiral matter of the Standard/Grand-Unified models. If such a goal were achieved, it would provide an important link between M-theory and realistic particle physics.

The aim of this paper is to address a specific aspect of this program. We focus on the systematic study of the supersymmetry conditions for general fluxes of massive Type IIA string theory, that yield four-dimensional $\mathcal{N} = 1$ supersymmetric vacua. This study is part of the program that aims at shedding light on the structure of four-dimensional $\mathcal{N} = 1$ supersymmetric vacua of Type IIA string theory with fluxes, along with the explicit construction of D-brane configurations. [The past few years have seen a surge of activities in explicit constructions of four-dimensional string vacua with intersecting D6-branes on Type IIA orientifolds (or magnetized D-branes on the dual Type IIB orientifolds) [32, 33] with semi-realistic particle physics [34, 35, 36]. [For first $\mathcal{N} = 1$ supersymmetric constructions of that type see [37, 38].] Within this framework important consequences for particle physics are due to the appearance of chiral matter at the D6-brane intersection points in the internal space [39, 40].]

It turns out that the structure of the possible flux configurations in Type IIA theory is very rich and the constraints are not well understood. One should contrast this situation with that of four-dimensional $\mathcal{N} = 1$ supersymmetric solutions of Type IIB theory with fluxes, which is much better understood. See, [7] and further work [41, 42] as well as recent efforts [43, 44] to construct supersymmetric chiral models with (magnetized) branes and fluxes.

On the Type IIA side specific progress has nevertheless been made. In Ref. [18] the $\mathcal{N} = 1$ supersymmetric vacuum given by intersecting D6-branes in the presence of NS-NS three-form fluxes for the massive TypeIIA supergravity was studied. Background fluxes induce, via back reaction on the geometry, non-zero intrinsic torsion components or G -structures. The presence of NS-NS three-form fluxes breaks the $SU(3)$ structures to $SU(2)$ and the D6-branes intersect at angles of $SU(2)$ rotations; non-zero mass parameter corresponds to D8-brane configurations which are orthog-

onal to the common cycle of all D6-branes. The anomaly inflow indicates that the gauge theory on intersecting (massive) D6-branes is not chiral [18]. Recent work [19] (see also [13, 17, 21, 27]) that focuses on a relationship between the constraints on fluxes of M-theory and those of Type IIA theory provide additional insights into specific structure of the supersymmetric flux configurations in massless Type IIA string theory.

Most recently, in [23], further progress has been made in this direction by analyzing the general supersymmetry constraints for four-dimensional $\mathcal{N}=1$ supersymmetric vacua of massive Type IIA supergravity where the Killing spinor is a $SU(3)$ singlet, and thus the internal manifold has $SU(3)$ structure. The result turned out to be extremely constraining, with the internal geometry corresponding to nearly-Kähler manifolds, with all the allowed fluxes proportional to the mass parameter, and the dilaton determined by a ratio of the (quantized) fluxes. The four-dimensional negative cosmological constant is also determined by the mass parameter and the dilaton field, and thus in the weak string coupling limit becomes small. There are explicit examples of nearly-Kähler manifolds with supersymmetric (intersecting) three cycles that the D6-branes can wrap; such backgrounds can therefore provide consistent string compactifications, *without orientifold planes*, where the non-Abelian chiral sector of the theory arises from the intersecting D6-branes wrapping the supersymmetric three-cycles, while the compactification moduli and the dilaton are fixed due to the turned on fluxes.

In this paper we advance a number of aspects of flux compactifications for supersymmetric vacua of massive Type IIA supergravity theory. We develop general techniques to study effectively the supersymmetry conditions with the most general spinor Ansatz in the case of both, $SU(3)$ and $SU(2)$ structures (Sections 2 and 3, respectively). In Sections 4 and 5 we explicitly derive the conditions on fluxes and geometry for $SU(3)$ and $SU(2)$ structures, respectively. In Section 4 we explicitly derive that the results of [23], provide a unique solution with the $SU(3)$ structure, whose internal geometry corresponds to the nearly-Kähler manifold, and all the turned on fluxes and the four-dimensional negative cosmological constant are proportional to the mass parameter. In Section 5 we explicitly derive a solution with $SU(2)$ structure where the four-dimensional space is Minkowski and the internal space is conformally

flat one. The $SU(2)$ structure of such solutions singles out the T^2 direction and we assume that all the field and metric coefficients depend only on the T^2 fiber coordinates. This solution has very interesting implications: the flux vacuum with the internal space conformally flat (with orbifold and orientifold projections) allows for explicit constructions of intersecting D6-brane configurations for which the gauge and chiral spectrum can now be calculated explicitly, using conformal field theory techniques. Section 6 is further devoted to the study of important physics implications for the vacuum solutions with the $SU(3)$ structure, in particular the specific examples of the nearly-Kähler internal geometry with supersymmetric intersecting three-cycles that allows for the consistent vacuum solutions with chiral non-Abelian gauge sectors. In addition, we also highlight a possibility to address the vacuum selection with the positive cosmological constant within this framework.

2 Fluxes and supersymmetry transformation in massive type IIA string theory

In this Section we shall spell out our notation, the relationships between the gauge potential and field strengths and the form of supersymmetry transformations in massive Type IIA supergravity theory. The notation and conventions are primarily following the work of Romans [45]; our notations are also explained in [18].

Bosonic fields In massive Type IIA string theory, the NS-NS 2-form and the RR 1-form potential combine into a gauge invariant (massive) 2-form given by (for convention see [46])

$$F = mB + dC_1 \quad (2.1)$$

and the 4-form becomes

$$G = dC_3 + \frac{1}{2m} F \wedge F \quad (2.2)$$

where C is the RR 3-form potential. Due to the Chern-Simons terms, both forms are not closed but

$$dF = mH \quad , \quad dG = F \wedge H \quad . \quad (2.3)$$

For the sake of notational simplicity we suppressed a subscript indicating the n -index

of a specific n -form. In the massless limit ($m = 0$) one gets

$$F^{(0)} = dC_1 \quad , \quad G = dC_3 + F^{(0)} \wedge B = d(C_3 + C_1 \wedge B) + C_1 \wedge H$$

Note, only in the massless case the 2-form $F = F^{(0)} = dC_1$ is exact.

Supersymmetry transformations Unbroken supersymmetry requires the existence of at least one Killing spinor ϵ , which is fixed by the vanishing of the fermionic supersymmetry transformations [for a purely bosonic configuration, the variations of the bosons vanish trivially]. These variations have been first setup for massive type IIA supergravity in Einstein frame [45], but we will use the string frame and the fermionic variations become [46]

$$\begin{aligned} \delta\psi_M &= \left\{ D_M + \frac{1}{8} H_M \Gamma_{11} + \frac{1}{8} e^\phi \left[m \Gamma_M + F \Gamma_M \Gamma_{11} + G \Gamma_M \right] \right\} \epsilon , \\ \delta\lambda &= \left\{ \partial\phi + \frac{1}{12} H \Gamma_{11} + \frac{1}{4} e^\phi \left[5m + 3F \Gamma_{11} + G \right] \right\} \epsilon . \end{aligned} \quad (2.4)$$

and we used the abbreviations

$$\partial \equiv \Gamma^M \partial_M , \quad H = H_{PQR} \Gamma^{PQR} , \quad H_M = H_{MPQ} \Gamma^{PQ} , \quad \text{etc.} \quad (2.5)$$

Apart from the differential forms that we introduced already, the mass parameter is denoted by m and ϕ is the dilaton. In type IIA supergravity, the Killing spinor ϵ is Majorana and can be decomposed into two Majorana-Weyl spinors of opposite chirality. The massless case can be lifted to M-theory and this Majorana spinor becomes the 11-dimensional Killing spinor. We will come back to our spinor convention below.

Ansatz for the Metric and field strength We are interested in compactifications on to a 4-d spacetime that is either flat or anti deSitter, i.e. up to warping the 10-d space time factorizes $M_{10} = X_{1,3} \times Y_6$ and we write the metric Ansatz as

$$ds^2 = e^{2A(y)} \left[g_{\mu\nu} dx^\mu dx^\nu + h_{mn}(y) dy^m dy^n \right] . \quad (2.6)$$

where $g_{\mu\nu}$ is either flat or AdS_4 and h_{mn} is the metric on Y_6 and the warp factors depend only on the coordinates of the internal space. An especially interesting question, which we will address in more detail below, are the constraints that allow for a flat internal/external space. Consistent with this metric Ansatz is the assumption that the fluxes associated with the forms F and H have non-zero components only in

the internal space Y_6 whereas G may have in addition a Freud-Rubin parameter λ :

$$\begin{aligned} F &= \frac{1}{2} F_{mn} dy^m \wedge dy^n \quad , \quad H = \frac{1}{3} H_{mnp} dy^m \wedge dy^n \wedge dy^p \quad , \\ G &= \lambda dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 + \frac{1}{4} G_{mnpq} dy^m \wedge dy^n \wedge dy^p \wedge dy^q \quad . \end{aligned} \quad (2.7)$$

Note, all forms as well as the warp factor and the dilaton are in general functions of coordinates y^m of the the internal space. With these Ansätze the gravitino variation splits into an external and internal part and with $D_M = \nabla_M + \frac{1}{2} \Gamma_M^N \partial_N A$, we find for the variations in (2.4)

$$\begin{aligned} 0 &= \left[\frac{1}{2} e^A \Gamma^\mu \nabla_\mu + \partial A + \frac{1}{4} e^{\phi+A} \left(m + \underline{F} \Gamma_{11} + \underline{G} - 4 \underline{\lambda} \Gamma_{0123} \right) \right] \epsilon \quad , \\ 0 &= \left[\nabla_m + \frac{1}{8} \underline{H}_m \Gamma_{11} - \frac{1}{4} e^A \Gamma_m (\Gamma^\mu \nabla_\mu) \right. \\ &\quad \left. - \frac{1}{2} e^{\phi+A} \left(\underline{F}_m \Gamma_{11} + 2 \underline{G}_m - 2 \underline{\lambda} \Gamma_{0123} \Gamma_m \right) \right] \epsilon \quad , \\ 0 &= \left[\partial \phi + \frac{1}{12} \underline{H} \Gamma_{11} + \frac{1}{4} e^{\phi+A} \left(5m + 3 \underline{F} \Gamma_{11} + \underline{G} + 4 \underline{\lambda} \Gamma_{0123} \right) \right] \epsilon \end{aligned} \quad (2.8)$$

where (∇_μ, ∇_m) are the covariant derivatives with respect to the metrics $(g_{\mu\nu}, h_{m,n})$ and we defined the rescaled forms

$$\underline{F} \equiv e^{-2A} F \quad , \quad \underline{G} \equiv e^{-4A} G \quad , \quad \underline{H} \equiv e^{-2A} H \quad , \quad \underline{\lambda} \equiv e^{-4A} \lambda \quad (2.9)$$

In order to proceed, we decompose the Γ -matrices as usual

$$\begin{aligned} \Gamma^\mu &= \hat{\gamma}^\mu \otimes \mathbb{1} \quad , \quad \Gamma^{m+3} = \hat{\gamma}^5 \otimes \gamma^m \quad , \quad \Gamma^{11} = -\hat{\gamma}^5 \otimes \gamma^7 \quad , \\ \hat{\gamma}^5 &= i \hat{\gamma}^0 \hat{\gamma}^1 \hat{\gamma}^2 \hat{\gamma}^3 \quad , \quad \gamma^7 = i \gamma^1 \gamma^2 \gamma^3 \gamma^4 \gamma^5 \gamma^6 \quad . \end{aligned} \quad (2.10)$$

We use the Majorana representation so that Γ^{11} , $\hat{\gamma}^\mu$ are real and $\hat{\gamma}^5$, γ^7 and γ^m are imaginary and anti-symmetric and to avoid confusions, we hatted the 4-dimensional γ -matrices and use m, n, \dots for internal indices and Greek for external indices. In addition, the spinor has to be decomposed into an 4-d spinor and an internal spinor, which will discuss in more detail in the next section. If the external space is flat, the 4-d spinor is covariantly constant, but if want to include AdS_4 vacua, which appear naturally as supersymmetric vacua, the covariant derivative of the 4-d spinor gives essentially the superpotential. If we denote the 4-d Killing spinor with θ and if it is Weyl, we introduce the complex superpotential W by

$$\nabla_\mu \theta = \hat{\gamma}_\mu \bar{W} \theta^* \quad . \quad (2.11)$$

As integrability constraint, one finds that the 4-d space time is anti-deSitter with $\Lambda = -|W|^2$ as the 4-d (negative) cosmological constant.

3 Killing spinors and G -structures

As for the bosonic fields, also the 10-d spinors have to be decomposed into an external and internal part and the general 10-d Killing spinor can be expanded in all independent internal and external spinors so that we can write in general

$$\epsilon = \theta_i \otimes \eta_i + cc \quad (3.1)$$

where θ_i and η_i are the four- and six-dimensional spinors, respectively. Recall, we use the convention that ϵ is a general Majorana spinor, not necessarily Weyl, and this spinor is now expanded in all independent internal and external spinors, where all internal spinors are chosen to be Weyl and the external are in general Dirac. Since these spinors have to be singlets under the structure group G , this most general case highly restricts the geometry of the internal space; e.g. only the flat space or the spheres can support four independent internal Weyl spinors. Note, in 11 dimensions the Majorana Killing spinor ϵ is expanded in up to eight internal and external Majorana spinors, which in the 10 dimensional spinor Ansatz are combined into internal Weyl and external Dirac spinors.

Since the spinors are singlets under this group, the resulting differential forms constructed as fermionic bi-linears are singlets under the structure group and define G -structures. We are interested in the two cases: (i) of $SU(3)$ structures and (ii) of $SU(2)$ structures. In the first case, only one singlet spinor η can appear, which implies for our Ansatz that: $\eta_i = a_i \eta$, with some complex coefficients a_i . The resulting 4-d spinor: $\theta = a_i \theta_i$ is one Dirac spinor, which can be written in terms of two Weyl or two Majorana spinors. If there are no further constraints on these two spinors, we obtain an $\mathcal{N}=2$, $D=4$ vacuum. Note, if there are no fluxes, the structure group determines the holonomy of the space and as expected an $SU(3)$ holonomy space leaves eight supercharges unbroken. For the $SU(2)$ case we can define two singlet spinors on the internal space, which can be combined into one doublet and also the 4-d spinors can be combined into one doublet of two Dirac spinors. With no fluxes, the holonomy is $SU(2)$ and hence the space factorize into $Y_6 = T_2 \times X_4$, where T_2 is flat and X_4 has $SU(2)$ holonomy, i.e. in the compact case it is K3 and the vacuum has $\mathcal{N}=4$ supersymmetry. We are of course interested in $\mathcal{N}=1$ vacua and therefore we have to break further supersymmetry by imposing a constraint on the 4-

d Dirac spinors, which can be understood as a flux-induced (partial) supersymmetry breaking. Actually, non-zero fluxes correspond to specific torsion components and the corresponding relations between fluxes and torsion will be derived in the next section. But let us now discuss in more detail the SU(3) and SU(2) case.

SU(3) structure

In this case, we have a single internal spinor η and the external spinor can be any Dirac spinor, which can be decomposed into two Weyl spinors of opposite chirality. Therefore, the spinor Ansatz becomes

$$\epsilon = c_i \theta_i \otimes \eta + cc$$

with the complex coefficients c_i ($i = 1, 2$) and $\hat{\gamma}^5 \theta_i = (\sigma_3 \eta)_i$ where σ_3 is third Pauli matrix³. In our notation the complex conjugate spinors θ_i^* have opposite chiralities and hence the truncation to an $\mathcal{N}=1$ vacuum is done by

$$\theta_i^* = (\sigma_1 \theta)_i$$

which is consistent with the Weyl property. Setting $\theta_1 = \theta$, the spinor Ansatz for an $\mathcal{N}=1$ vacuum becomes

$$\epsilon = (a\theta + b\theta^*) \otimes \eta + cc = \theta \otimes (a\eta + b^* \eta^*) + cc . \quad (3.2)$$

There are two special cases: if $ab = 0$ the 4-d spinor is Weyl and if $b = a^*$ we have a Majorana spinor.

On the other hand, the SU(3) singlet spinor is Weyl and we choose

$$\eta = \frac{1}{\sqrt{2}} (\mathbb{1} + \gamma^7) \eta_0 . \quad (3.3)$$

with η_0 being a constant spinor. Being an SU(3) singlet spinor, η satisfies the projectors

$$\begin{aligned} (\gamma_m + i J_{mn} \gamma^n) \eta &= 0 \\ (\gamma_{mn} - i J_{mn}) \eta &= \frac{i}{2} \Omega_{mnp} \gamma^p \eta^* , \end{aligned} \quad (3.4)$$

$$(\gamma_{mnp} - 3i J_{[mn} \gamma_{p]}) \eta = i \Omega_{mnp} \eta^* .$$

³With the usual definition: $\sigma_1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} & -i \\ i & \end{pmatrix}$, $\sigma_3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$.

where the complex structure and the holomorphic 3-form are introduced by

$$\eta^\dagger \gamma_{mn} \eta = i J_{mn} \quad , \quad \eta \gamma_{mnp} \eta = i \Omega_{mnp} \quad (3.5)$$

[with $1 = \eta^\dagger \eta$]. Note, these are the only differential forms that can be constructed from a single chiral spinor and for non-zero fluxes they are *not* covariantly constant nor closed and this failure is related to non-vanishing intrinsic torsion components. Following the literature [47, 12, 14, 15], one introduces five classes \mathcal{W}^i by

$$\begin{aligned} dJ &= \frac{3i}{4} (\mathcal{W}_1 \bar{\Omega} - \bar{\mathcal{W}}_1 \Omega) + \mathcal{W}_3 + J \wedge \mathcal{W}_4 , \\ d\Omega &= \mathcal{W}_1 J \wedge J + J \wedge \mathcal{W}_2 + \Omega \wedge \mathcal{W}_5 \end{aligned} \quad (3.6)$$

with the constraints: $J \wedge J \wedge \mathcal{W}_2 = J \wedge \mathcal{W}_3 = \Omega \wedge \mathcal{W}_3 = 0$. Depending on which torsion components are non-zero, one can classify the geometry of the internal space. E.g., if only $\mathcal{W}_1 \neq 0$ the space is called nearly Kähler, for $\mathcal{W}_2 \neq 0$ almost Kähler, the space is complex if $\mathcal{W}_1 = \mathcal{W}_2 = 0$ and it is Kähler if only $\mathcal{W}_5 \neq 0$.

SU(3) decomposition of fluxes

This decomposition is done by employing the holomorphic projector: $\frac{1}{2}(1 \pm iJ)$ to distinguish between holomorphic and anti-holomorphic indices. We will indicate this by the labels at the forms, but let us stress that this makes sense only locally. Since the internal space is in general not a complex manifold, one cannot introduce global holomorphic forms.

On the 6-dimensional internal space Y_6 , the 4-form G and the 2-form F have 15 and the 3-form 20 components, which decomposes as follows

$$[G] = \mathbf{8} + \mathbf{1} + \mathbf{3} + \bar{\mathbf{3}} = [G^{(2,2)}] + [G_0^{(2,2)}] + [G^{(3,1)}] + [G^{(1,3)}] \quad (3.7)$$

with the singlet $G_0^{(2,2)} = G_{mnpq} J^{mn} J^{pq}$ and the vector representations⁴: $G^{(3,1)} = \Omega \lrcorner G$ and its complex conjugate: $G^{(1,3)} = \bar{\Omega} \lrcorner G$; the remaining components comprise the (2,2)-forms obeying: $G^{(2,2)} \wedge J = 0$.

Similarly, the components of the 2-form F decomposes as

$$[F] = \mathbf{8} + \mathbf{1} + \mathbf{3} + \bar{\mathbf{3}} = [F^{(1,1)}] + [F_0^{(1,1)}] + [F^{(2,0)}] + [F^{(0,2)}] \quad (3.8)$$

⁴Here and in the following we use the convention: $(\Omega \lrcorner G)_m \equiv \Omega_{abc} G^{abc}_m$.

with the singlet: $F_0^{(1,1)} = J \lrcorner F$, the vectors $F^{(2,0)} = F \lrcorner \Omega$ and its complex conjugate. The remaining components represent an adjoint of $SU(3)$ satisfy: $F^{(1,1)} \wedge J \wedge J = 0$.

Finally, the 3-form decomposes as

$$\begin{aligned} [H] &= \mathbf{6} + \bar{\mathbf{6}} + \mathbf{3} + \bar{\mathbf{3}} + \mathbf{1} + \bar{\mathbf{1}} \\ &= [H^{(2,1)}] + [H^{(1,2)}] + [H_0^{(2,1)}] + [H_0^{(1,2)}] + [H^{(3,0)}] + [H^{(0,3)}] \end{aligned} \quad (3.9)$$

where the singlets are now: $H^{(3,0)} = \Omega \lrcorner H$ and its complex conjugate; the two vectors $H_0^{(2,1)}$ and $H_0^{(1,2)}$ are the two holomorphic projections of the (real) vector: $J \lrcorner H$ and the $\mathbf{6} + \bar{\mathbf{6}}$ are the primitive (2,1) and (1,2) forms fulfilling $H \wedge J = 0$.

SU(2) structure

If the structure is broken to $SU(2)$, one finds two singlet spinors η_k and we choose them of opposite chirality, i.e.

$$\gamma^7 \eta_k = (\sigma_3 \eta)_k \quad (3.10)$$

with: $(\sigma_3 \eta)_k \equiv (\sigma_3)_k^l \eta_l$. In addition there two external Dirac spinors θ_i and in order to obtain an $\mathcal{N}=1$ vacuum we have to impose projectors. As for the $SU(3)$ case, we can first truncate each Dirac spinor into one Weyl spinor and write as in (3.2)

$$\epsilon = (a_1 \theta_1 + b_1 \theta_1^*) \otimes \eta_1 + (a_2 \theta_2 + b_2 \theta_2^*) \otimes \eta_2 + cc \quad (3.11)$$

where a_i, b_i are complex coefficients. Now, the truncation to $\mathcal{N}=1$ is given by the following projectors obeyed by the 4-d spinors

$$\theta_i = \hat{\gamma}^5 \theta_i \quad , \quad \theta_i = (\sigma_1 \theta)_i$$

so that both spinors have the same chirality and moreover $\theta_2 = \theta_1 \equiv \theta$ [σ_1 can also be replaced by σ_2]. Thus, ϵ becomes

$$\epsilon = \theta \otimes (a_i \eta_i + b_i^* \eta_i^*) + cc = (\theta a_i + \theta^* b_i) \otimes \eta_i + cc . \quad (3.12)$$

Here, θ is a single Weyl spinor, but if we set in (3.11): $a_1 = b_1 = c_1$ and $a_2 = -b_2 = i c_2$, this spinor becomes: $\epsilon = c_1 \theta_1^M \otimes \eta_1 + i c_2 \hat{\gamma}^5 \theta_2^M \otimes \eta_2 + cc$, with the 4-d Majorana spinor $\theta_i^M = \theta_i + \theta_i^*$. For this Majorana case, the truncation to $\mathcal{N}=1$ vacuum corresponds to the relations $i \hat{\gamma}^5 \theta_1^M = \theta_2^M$ between the two Majorana spinors θ_1^M and θ_2^M .

The internal spinors η_i have opposite chirality and one of them can be taken as the SU(3) singlet, say η_1 . The other can then be introduced by means of a vector field v_m

$$\eta_2 = v \eta_1 \equiv v_m \gamma^m \eta_1 \quad \text{with : } |v| = 1 .$$

With these spinors, we define the following differential forms

$$\Lambda_{kl}^{(n)} = \eta_k^\dagger \gamma^{(n)} \eta_l \quad , \quad \Sigma_{kl}^{(n)} = \eta_k^T \gamma^{(n)} \eta_l \quad (3.13)$$

where $\gamma^{(n)} \equiv \gamma_{m_1 m_2 \dots m_n}$. Each of this form is a 2×2 matrix and in the following we will use a matrix notation in terms of Pauli matrices. The spinors are normalized and chiral so that

$$\Lambda^{(0)} = \mathbb{1} \quad , \quad \Sigma^{(0)} = 0$$

We recover the SU(3) expressions in (3.5) by setting $v = 0$

$$\text{SU(3) case : } \quad \Lambda^{(2)} = i J \quad , \quad \Lambda^{(4)} = -\frac{1}{2} J \wedge J \quad , \quad \Sigma^{(3)} = i \Omega$$

which does not allow for a vector. But in the SU(2) case one can construct two vectors, which can be combined into one complex vector: $\eta_1^\dagger \gamma_m \eta_2 = v_m + i J_m{}^n v_n \equiv 2v_m^{(+)}$. With the definition

$$u^m = J^m{}_n v^n$$

we find the following useful relations ($v \equiv v_m \gamma^m$ etc.)

$$\eta_k = v (\sigma_1)_k{}^l \eta_l = u (\sigma_2)_k{}^l \eta_l \quad \text{or : } \quad (v - iu) \eta_1 = (v + iu) \eta_2 = 0 \quad (3.14)$$

and hence we get for the 1-forms

$$\Lambda^{(1)} = v \sigma_1 - u \sigma_2 \quad , \quad \Sigma^{(1)} = 0 .$$

With this 1-form, the spinors η_k satisfy the projectors

$$(\mathbb{1} + \frac{1}{2} \Lambda_m \gamma^m) \eta^\star = 0 \quad , \quad \Lambda_m \gamma^m \eta = 0 .$$

The existence of the (complex) vector implies, that the 6-d internal space is locally a complex line bundle over a 4-d base space. We find for the 2-forms

$$\Lambda^{(2)} = i \mathbb{1} J^{(0)} + i \sigma_3 v \wedge u \quad , \quad \Sigma^{(2)} = \sigma_2 \hat{\Omega} . \quad (3.15)$$

Similar to the $SU(3)$ case, $J^{(0)}$ is an almost complex structure on the base and can be used to project onto holomorphic and anti-holomorphic components. In addition, $\hat{\Omega}_{ab} \equiv \Omega_{abc}v^c$ is a (2,0)-form and its complex conjugate is (0,2)-form. If the spinors would be covariantly constant, as in the absence of fluxes, these forms would identify the base as an hyper-Kähler space. Note, we identified the symplectic 2-form by the relation to the $SU(3)$ case, but actually all three 2-forms are on equal footing and one can also pick another one. The two spinors satisfy moreover the relations

$$\hat{\Omega}^{ab}\gamma_{ab}\eta^\star = -8\sigma_2\eta \quad , \quad \hat{\Omega}^{ab}\gamma_{ab}\eta = 0 \quad , \quad J_{ab}^{(0)}\gamma^{ab}\eta = 4i\eta \quad (3.16)$$

[recall, we are using a matrix notation and we have the doublet spinor $\eta \equiv (\eta_1, \eta_2)$]. There is no 3-form on the base and hence all 3-forms have to have one leg in the fiber and we find

$$\begin{aligned} \Lambda^{(3)} &= \Lambda^{(1)} \wedge \Lambda^{(2)} = i(\sigma_1 v - \sigma_2 u) \wedge J^{(0)} \quad , \\ \Sigma^{(3)} &= -\Lambda^{(1)} \wedge \Sigma^{(2)} = -(\mathbb{1}u - i\sigma_3 v) \wedge \hat{\Omega} \quad . \end{aligned} \quad (3.17)$$

The 4- and 5-forms are dual to the 2- and 1-forms. where one has however to take into account the multiplication with σ_2 . For the 4-forms one finds

$$\Lambda^{(4)} = -\mathbb{1} J^{(0)} \wedge J^{(0)} + i\sigma_3 u \wedge v \wedge J^{(0)} \quad , \quad \Sigma^{(4)} = \sigma_1 u \wedge v \wedge \hat{\Omega}_{ab} \quad (3.18)$$

and the 5-forms read

$$\Lambda^{(5)} = \Lambda^{(1)} \wedge \Lambda^{(4)} = (v\sigma_1 - u\sigma_2) \wedge \Lambda^{(4)} \quad , \quad \Sigma^{(5)} = 0$$

As in the $SU(3)$ case, these forms are not closed if fluxes are present, which again is related to non-vanishing torsion components.

For the $SU(3)$ case the geometry was described by the almost complex structure J and the holomorphic 3-form Ω . Here in the $SU(2)$ case, the 6-d geometry is fixed by the triplet $(v, J^{(0)}, \hat{\Omega}^{(2,0)})$. If all fluxes vanish, the $SU(3)$ case corresponds to a Calabi-Yau space whereas the $SU(2)$ case yields a factorization of the internal space into $T^2 \times K3$ (if we assume compactness). As we will see below, the $SU(2)$ case allows for much more general fluxes than the highly constrained $SU(3)$ case.

Summary

When truncated to an $\mathcal{N}=1$ vacuum, both spinor Ansätze can be written as

$$\epsilon = \theta \otimes \chi + \theta^\star \otimes \chi^\star$$

where χ is not chiral spinor but: $\chi = a\eta + b^*\eta^*$ for the SU(3) case and: $\chi = a_i\eta_i + b_i^*\eta_i^*$ for the SU(2) case. If we introduce the superpotential as in (2.11) we find

$$\Gamma^\mu \nabla_\mu \epsilon = -4W \theta \otimes \chi^* + 4\bar{W} \theta^* \otimes \chi .$$

The 4-dimensional spinor θ is chiral and therefore all terms coming with θ and θ^* have to vanish separately. If we collect all terms $\mathcal{O}(\theta)$, the variations (2.8) become

$$-\partial A \chi + 2e^A W \chi^* = \frac{1}{4}e^{\phi+A} \left[m - \underline{F} \gamma_7 + \underline{G} + 4i \underline{\lambda} \right] \chi , \quad (3.19)$$

$$-\left[\partial \phi - \frac{1}{12} \underline{H} \gamma^7 \right] \chi = \frac{1}{4}e^{\phi+A} \left[5m - 3 \underline{F} \gamma^7 + \underline{G} - 4i \underline{\lambda} \right] \chi , \quad (3.20)$$

$$\left[\nabla_m + \frac{1}{8} \underline{H}_m \gamma^7 \right] \chi - e^A W \gamma_m \chi^* = -e^{\phi+A} \left[\frac{1}{2} \underline{F}_m \gamma^7 - \underline{G}_m - i \underline{\lambda} \gamma_m \right] \chi . \quad (3.21)$$

The complex conjugate of these equations yield the terms coming together with θ^* .

4 Fluxes and geometry for SU(3) structures

Now we have all the tools to explore in detail the Killing spinor equations. With SU(3) structures, the internal spinor reads

$$\chi = a\eta + b^*\eta^* .$$

A special case is if χ is chiral, i.e. a or b is zero and the 10-dimensional spinor ϵ is Majorana-Weyl. Another special case would be $b^* = a$, where the 4-d spinor is Majorana. Let us discuss the different cases separately.

Weyl case

Setting $b = 0$ (or $a = 0$), the lhs and rhs of the equations (3.19) – (3.21) have opposite chirality and have to vanish separately! One finds the same constraints as the one derived for $m = 0$ in [21, 19], namely: $F = G = H^{(3,0)} = 0$ ⁵ and $d\phi = H \lrcorner J^2 \equiv H \lrcorner (J \wedge J)$. The vanishing of F and G follows from the internal variation (3.21), which yields after using the expression in (3.4)

$$\underline{G}^{(3,1)} = 0 \quad , \quad \underline{F}_m{}^l J_{nl} + 6 \underline{G}_{mnpq} J^{pq} + 4 \underline{\lambda} \delta_{mn} = 0 . \quad (4.1)$$

⁵Since H is a real form, this implies also that $H^{(0,3)} = 0$.

By taking the trace, the singlet $F_0^{(1,1)}$ is canceled by $\underline{\lambda}$ and all other components have to vanish

$$F \sim \lambda J \quad , \quad G_{mnpq} = 0.$$

Note, in real notation, $F^{(1,1)}$ satisfies: $(1 \pm iJ) \cdot F^{(1,1)} \cdot (1 \mp iJ) = 0$ yielding that $F^{(1,1)} \cdot J$ is a symmetric matrix. From the rhs of the external variation (3.19), we find a complex constraints which set the mass to zero and another constraint on the Freud-Rubin parameter $\underline{\lambda}$ and $F_0^{(1,1)}$ which is in contradiction with the relation derived from the internal variation. Therefore one has to infer: $\underline{\lambda} = \lambda = 0$, $m = 0$. Contracting the lhs of (3.19) with η^T and η^\dagger one gets moreover that: $W = 0$ and hence: $dA = 0$. So, all RR fields have to be zero, but the NS- H -field can still be non-zero. Setting $A = 0$, we find from the dilatino variation (3.20)

$$H \lrcorner J^2 = d\phi \quad , \quad H \lrcorner \Omega = 0 \quad (4.2)$$

i.e. the dilaton is fixed by $H_0^{(2,1)}$. These H -field components enter the differential equation fixing the Killing spinor and by investigating the torsion classes one finds that the internal space is in general non-Kählerian [47, 15].

In summary, if the 10-d spinor is Majorana-Weyl or equivalently if $ab = 0$ in our spinor, the mass and all RR-fields have to vanish:

$$F_{mn} = G_{mnpq} = W = dA = m = 0 \quad (4.3)$$

and only the fields from the NS-sector can be non-trivial. The holomorphic part $H^{(3,0)}$ has to vanish, $H_0^{(2,1)}$ fixes the dilaton and the internal space is non-Kählerian. One might have expected this result, because the NS-sector is common to all string models, and a common solution can only be described by one 10-d Majorana-Weyl (Killing) spinor. An explicit example that solves these equations is the NS5-brane supergravity solution, but there are also other examples [1, 15].

Majorana case

Another interesting case is given by $a = b^*$, where 4-dimensional spinor $a\theta + b\theta^*$ in (3.12) is Majorana. This implies that the 10-d spinor cannot be Weyl and hence corresponds to the more generic situation for type IIA models. In this case

$$\chi = a(\eta + \eta^*)$$

and we can separate the real and imaginary part of the equations (3.19) – (3.21), which will give us two sets of equations. This case has been discussed already in [23], but since our notation here differ, let us summarize this case.

If we define $a = re^{i\alpha}$ and $W = e^{2i\alpha}(W_1 + iW_2)$ with W_1 and W_2 real, the first set becomes

$$e^A W_1 \eta = \frac{1}{8} e^{\phi+A} (m + \underline{G}) \eta \quad (4.4)$$

$$- \underline{H} \eta^* = 3 e^{\phi+A} (5m + \underline{G}) \eta \quad (4.5)$$

$$i(\partial_m \alpha) \eta^* + \frac{1}{8} \underline{H}_m \eta^* + e^A W_1 \gamma_m \eta = e^{\phi+A} \underline{G}_m \eta \quad (4.6)$$

With the formulae in (3.4), the first two equations give: $G^{(1,3)} = 0$ and $J \lrcorner H = 0$ (i.e. $H \wedge J = 0$); in the last equation the singlets give the constraint

$$d\alpha = 0 \quad , \quad 6 W_1 - 3 H_0 = e^\phi G_0 \quad (4.7)$$

with

$$G = G_0 J \wedge J \quad , \quad H = H_0 \text{Im} \Omega \quad (4.8)$$

which are the singlet components under an $SU(3)$ decomposition. With (4.4) and (4.5) we find for these singlets

$$W_1 = -\frac{m}{10} \quad , \quad G_0 = \frac{m}{20} \quad , \quad H_0 = -\frac{2m}{5} e^\phi \quad . \quad (4.9)$$

The remaining components in G_m and H_m in (4.6) cannot cancel, due to the different holomorphic structure and therefore have to vanish. Note, all components of G and H can be obtained from $\hat{G}_{mn} \equiv G_{mnpq} J^{pq}$ and $\hat{H}_{mn} \equiv H_{mpq} \Omega^{pq}_n$ by different chiral projections. E.g. the (1,1) part in \hat{H}_{mn} are the $\mathbf{1} + \bar{\mathbf{1}}$ (ie. $H^{(3,0)}$ and $H^{(0,3)}$) and the (2,0) + (0,2) components are in total $3 \times 3 = 9$ complex or $\mathbf{18} = \mathbf{6} + \mathbf{3} + \bar{\mathbf{6}} + \bar{\mathbf{3}}$ real components; see also (3.9).

The remaining equations obtained from (3.19) – (3.21) are

$$- \partial A \eta + 2i e^A W_2 \eta^* = \frac{1}{4} e^{\phi+A} (\underline{F} + 4i \underline{\lambda}) \eta^* \quad (4.10)$$

$$- \partial \phi \eta = \frac{1}{4} e^{\phi+A} (3 \underline{F} - 4i \underline{\lambda}) \eta^* \quad (4.11)$$

$$(\hat{\nabla}_m + \partial_m r) \eta + i \gamma_m e^A W_2 \eta^* = \frac{1}{2} e^{\phi+A} (\underline{F}_m + 2i \underline{\lambda} \gamma_m) \eta^* \quad (4.12)$$

These are differential equations for the warp factors, the dilaton and additional constraints on the $W_{1/2}$, λ as well as $F_0^{(1,1)}$. One finds

$$F_0 = -\frac{2}{9} \lambda e^{-4A} , \quad W_2 = -\frac{2}{3} \lambda e^{\phi-3A} , \quad (4.13)$$

and moreover

$$\begin{aligned} \nabla_m \hat{\eta} &= \frac{1}{2} e^{\phi+A} (F_{mn} \gamma^n + i \frac{32}{9} \underline{\Delta} \gamma_m) \hat{\eta}^* , \\ \partial_m A &= -\frac{i}{8} e^{\phi+A} \Omega_{mpq} \underline{F}^{pq} \end{aligned} \quad (4.14)$$

with $\hat{\eta} = e^r \eta$.

But it is not enough to consider the supersymmetry variation, one has also to ensure that the 10-d equations of motion for G and H are solved as well as the Bianchi identities. The only solution that we found requires [23]

$$d\phi = dA = 0 , \quad F = F_0 J \quad (4.15)$$

with constant G_0 , H_0 given by (4.9) and F_0 by (4.13). Note, A being constant, they can be scaled away and we can set $A = 0$ from the very beginning. Thus, eq. (4.9) implies that the dilaton is fixed by the ratio of the (quantized) fluxes

$$e^\phi = -\frac{H_0}{8G_0} . \quad (4.16)$$

The differential equation for the spinor becomes finally

$$\nabla_m \eta = i \frac{17}{9} e^\phi \lambda \gamma_m \eta^* \quad (4.17)$$

which identifies the internal space as a nearly Kähler manifold, which is Einstein but neither complex nor Kähler! It can also be written as

$$0 = \hat{\nabla}_m \eta = \left[\nabla_m + \frac{34}{9} e^\phi \operatorname{Re} \Omega_{mpq} \gamma^{pq} \right] \eta$$

and hence the spinor is covariantly constant with respect to the Bismut connection (its holonomy is in $SU(3)$). It is straightforward to verify that in this case only $\mathcal{W}_1 \neq 0$ whereas all other torsion classes in (3.6) vanish. In fact, $dJ = \frac{17}{18} \lambda e^\phi \operatorname{Im} \Omega$, $d \operatorname{Re} \Omega \sim J \wedge J$ which ensures $dF = mH$, $dH = 0$ and $d^* H \sim G \wedge G$. This also fixes the Freud-Rubin parameter in terms of the mass:

$$\sqrt{\frac{85}{2}} \lambda = 9m .$$

In the limit of vanishing mass, our solution becomes trivial, i.e. all fluxes vanish and the internal space becomes Calabi-Yau. There is however no direct limit to massless configurations related to intersecting D6-branes, for which the torsion classes are: $\mathcal{W}_1 = \mathcal{W}_3 = 0$, but $\mathcal{W}_2, \mathcal{W}_4, \mathcal{W}_5 \neq 0$ [17].

The differential equation of the spinor can be solved by a constant spinor if one imposes first order differential equations on the Vielbeine e^n

$$\omega^{pq} J_{pq} = 0, \quad \omega^{pq} \Omega_{pq}{}^n = -\frac{\lambda}{9} e^\phi e^n \quad (4.18)$$

where $\omega^{pq} \equiv \omega_m^{pq} dy^m$ are the spin-connection 1-forms. Therefore, 6-d nearly Kähler spaces can be seen as a weak SU(3)-holonomy space, which like Calabi-Yau spaces have, e.g., a vanishing first Chern class. Their close relationship to special holonomy spaces comes also due to the fact that the cone over nearly Kähler 6-manifolds become a G_2 -holonomy spaces [48], defined by a covariantly constant spinor. This can be verified by multiplying (4.18) with J from the right and identifying the rhs as the spin connection ω^{7n} . Note, the spin connection 1-form of G_2 holonomy spaces satisfy $\omega^{MN} \varphi_{MNP} = 0$, where φ_{MNP} is the G_2 -invariant 3-form. It is hence straightforward to construct nearly Kähler spaces starting from G_2 holonomy spaces and the almost Kähler form, that defines our vacuum completely, is then given by $J_{mn} = \varphi_{mn7}$.

Generic case

If the complex coefficients a and b are generic, the solution becomes more involved. Recall, in the cases discussed before either the mass parameter had to vanish or the external space cannot be flat and one might wonder whether this generic case allows for flat space vacuum with non-zero mass parameter and therefore go beyond solutions obtained from M-theory.

We need to consider only the singlets of the fluxes, because they are related to the mass parameter or generate a non-zero cosmological constant (or superpotential). If $F = F_0 J + \dots$ and $G = G_0 J \wedge J + \dots$ and setting $W = 0$, we find from (3.19) that $F_0 = 0$ and $m \sim G_0$. Then, the dilaton variation in (3.20) gives, up to a real coefficient, the equation

$$H \gamma^7 \chi = H(a\eta - b^* \eta^*) \sim m\chi = m(a\eta + b^* \eta^*) .$$

Using (3.4), we find for singlet parts of H : $(H \lrcorner \Omega) \sim i \frac{a}{b^*}$ and $(H \lrcorner \Omega^*) \sim -i \frac{b^*}{a}$. These equations are consistent only, if

$$|a|^2 = |b|^2 .$$

Hence, a and b differ only by a phase (ie. $a = re^{i\alpha}$, $b = re^{i\beta}$) and we can write the spinor Ansatz

$$\chi = a\eta + b^*\eta^* = re^{i\frac{\alpha-\beta}{2}} \left[\tilde{\eta} + \tilde{\eta}^* \right]$$

with $\tilde{\eta} = e^{-i\frac{\alpha+\beta}{2}}\eta$. This however, is equivalent to the Majorana case discussed before and therefore, by investigating the internal variation, we will find again, that non-trivial fluxes are only possible if the mass parameter vanishes or if the cosmological constant is non-zero.

5 Fluxes and geometry for SU(2) structures

In the case of SU(2) structures the internal spinor, entering (3.19) – (3.21), becomes

$$\chi = a_i \eta_i + b_i^* \eta_i^*$$

and for the sake of simplicity we will further drop the index i and keep in mind, that a and b are now vectors and η is a spinor doublet. Solutions with SU(2) structures are in general very involved, and at this stage we shall not discuss the most general case. In contrast with the unique solution that we found for SU(3) structures, we are now interested in finding a specific flux vacuum whose 4-d space-time is conformally Minkowski space and the internal space is conformally flat. Therefore, we will set in the following

$$W = \lambda = 0$$

and write the metric as

$$ds^2 = e^{2A}[-dt^2 + dx^i dx^i] + e^{2B}[dv^2 + du^2 + dz^1 d\bar{z}^1 + dz^2 d\bar{z}^2] \quad (5.1)$$

where ∂_u and ∂_v are the two (global) vectors that we introduced before. The internal metric becomes therefore: $h_{mn} = e^{2(B-A)}\delta_{mn}$ and in the definition of \underline{F} , \underline{G} and \underline{H} in (2.9) we have to replace: $A \rightarrow B$. The existence of a flat vacuum with an internal Calabi-Yau space has been suggested in [19], but it was unclear whether it can also

be made flat. As we will show now, there exist in fact such a vacuum and the fields can be given explicitly.

For our solution, the warp factors and the dilaton depend only on the coordinates u and v so that

$$A = A(v, u) \quad , \quad B = B(v, u) \quad , \quad \phi = \phi(v, u) \quad .$$

With the (massive) 2-form F also the H - and G -flux is fixed (we assume here that $C_3 = 0$). The component of F proportional to $u \wedge v$ does not contribute to the H -field and enter the equations (3.19) and (3.20) in the same way as the Freund-Rubin parameter λ (but with different factors). By including this component as well as the analogous component of G (ie. where two legs of the 4-form are along $u \wedge v$), we could not find other solutions and therefore we set these flux components to zero from the beginning. Therefore, let us consider a 2-form restricted to the 4-d base space spanned by the two complex coordinates (z^1, z^2) , which allows in total for six 2-forms: three are the $SU(2)$ singlets J_0 , $\text{Re}\hat{\Omega}$, $\text{Im}\hat{\Omega}$ and in real coordinates they can be chosen to be selfdual. In addition there are three anti-selfdual 2-forms, which are, in respect to the symplectic form J_0 , primitive and of (1,1)-type. Each set, the selfdual and anti-selfdual forms obey a quaternionic algebra and one finds that the 2-form components along each of the three selfdual and three anti-selfdual are equivalent and therefore we will pick just one selfdual and one anti-selfdual component and write

$$F = m e^{2B} \left[f_0 \text{Im}(dz^1 \wedge dz^2) + f_1 \text{Im}(dz^1 \wedge d\bar{z}^2) \right] \quad (5.2)$$

where the constant f_0 parameterize the $SU(2)$ singlet part and the f_1 parameter is related to a primitive (1,1)-part of F . Then, 3- and 4-form (recall $C_3 = 0$) become

$$\begin{aligned} H &= de^{2B} \wedge \left[f_0 \text{Im}(dz^1 \wedge dz^2) + f_1 \text{Im}(dz^1 \wedge d\bar{z}^2) \right] , \\ G &= \frac{1}{2m} F \wedge F = m e^{4B} (f_0^2 - f_1^2) vol_4 \end{aligned}$$

where vol_4 is volume form for the 4-d base base. In the discussion of the BPS eqs. (3.19) – (3.21) we will start with the last equation, which is the most constraining one. Because F and G have no components along the (u, v) space, we obtain the equations

$$\begin{aligned} \left[\frac{1}{2} \gamma_u \gamma^v \partial_v (B - A) - \frac{1}{8} \underline{H}_u \gamma^7 \right] \chi &= 0 , \\ \left[\frac{1}{2} \gamma_v \gamma^u \partial_u (B - A) - \frac{1}{8} \underline{H}_v \gamma^7 \right] \chi &= 0 . \end{aligned} \quad (5.3)$$

With the relations (3.16) and $\underline{H}_{npq} = \frac{1}{m} \partial_n B \underline{F}_{pq} + cycl$, one finds

$$\begin{aligned}\underline{H}_v \gamma^7 \chi &= 4f_0 \partial_v B [-ia\sigma_2\sigma_3\eta^* + ib^*\sigma_2\sigma_3\eta] , \\ \underline{H}_u \gamma^7 \chi &= 4f_0 \partial_u B [-ia\sigma_2\sigma_3\eta^* + ib^*\sigma_2\sigma_3\eta] , \\ \gamma_v \gamma^u \partial_u (B - A) \chi &= \partial_u (B - A) [-ia\sigma_3\eta + ib^*\sigma_3\eta^*] , \\ \gamma_u \gamma^v \partial_u (B - A) \chi &= \partial_v (B - A) [ia\sigma_3\eta - ib^*\sigma_3\eta^*] .\end{aligned}$$

Note, the f_1 part drops out here, because the SU(2) singlet spinor χ picks up only the SU(2)-singlet component of the 2-form F , which can be verified by employing the projectors in (3.4). With these expressions, the terms in (5.3) cancel, if the two complex vectors a and b^* obey

$$b^* = \pm(a\sigma_2) \quad (5.4)$$

[in the following we choose the “+”-sign] and A and B have to satisfy the equations

$$\begin{aligned}f_0 \partial_v B + \partial_u (B - A) &= 0 , \\ f_0 \partial_u B - \partial_v (B - A) &= 0 .\end{aligned} \quad (5.5)$$

Since these are the Cauchy-Riemann equations, B and $(B - A)$ can be combined into one holomorphic function depending on the complex coordinate $w = v + i u$.

Using also the relation (3.14) and $G_{rspq} \gamma^{rspq} = \frac{1}{2m} [(F_{rs} \gamma^{rs})(F_{pq} \gamma^{pq}) + 2F_{pq} F^{pq}]$, we find for the other terms in the BPS equations (3.19) and (3.20)

$$\begin{aligned}\partial A \chi &= \partial_v A [a\sigma_1\eta - b^*\sigma_1\eta^*] + \partial_u A [a\sigma_2\eta + b^*\sigma_2\eta^*] \\ \underline{F} \gamma^7 \chi &= 4m f_0 [-ia\sigma_2\sigma_3\eta^* + ib^*\sigma_2\sigma_3\eta] , \\ \underline{G} \chi &= 4m (f_1^2 - f_0^2) \chi , \\ \underline{H} \gamma^7 \chi &= -12f_0 \left(\partial_v B [b^*\eta + a\eta^*] + i\partial_u B [b^*\sigma_3\eta - a\sigma_3\eta^*] \right) .\end{aligned} \quad (5.6)$$

Note, the f_1 parameter enters only the 4-form $G = \frac{1}{2m} F \wedge F$, which is proportional to the volume form and both terms of F enter with the opposite sign. In the BPS equations these terms can cancel only if we impose another constraint on a , namely⁶

$$a = \sigma_1 a \quad (5.7)$$

⁶A more general constraint like: $a = \cos \alpha (\sigma_1 a) + \sin \alpha (\sigma_2 a)$ yields at the end the same results.

i.e. a is an eigenvector to σ_1 . This fixes the vector a and due to (5.4) also b , up an overall factor and the expression in (5.6) can be expressed in terms of the original spinor χ and we get

$$\begin{aligned}
\partial A \chi &= [\partial_v A + i\gamma^7 \partial_u A] \chi \\
\underline{F} \gamma^7 \chi &= 4m f_0 i \gamma^7 \chi , \\
\underline{G} \chi &= 4m (f_1^2 - f_0^2) \chi , \\
\underline{H} \gamma^7 \chi &= 12f_0 [\partial_u B + i\gamma^7 \partial_v B] \chi .
\end{aligned} \tag{5.8}$$

Therefore we find from (3.20)

$$\begin{aligned}
f_0 \partial_u B - \partial_v \phi &= e^{\phi+B} [\tfrac{5}{4} + (f_1^2 - f_0^2)] m , \\
f_0 \partial_v B + \partial_u \phi &= \tfrac{3}{4} e^{\phi+B} f_0 m
\end{aligned} \tag{5.9}$$

and in the external variation (3.19), we use the Cauchy-Riemann eqs. (5.5) to replace A by B and find

$$\begin{aligned}
f_0 \partial_u B - \partial_v B &= e^{\phi+B} [\tfrac{1}{4} + (f_1^2 - f_0^2)] m \\
f_0 \partial_v B + \partial_u B &= e^{\phi+B} m f_0
\end{aligned} \tag{5.10}$$

These two sets of equations fix the dilaton ϕ and B , which in turn, due to (5.5), fixes the warp factor A .

There is still one set of equations left, which are the remaining components of the internal variation (3.21) (we solved so far only the (u, v) -part). If we contract this equation with γ^m , we find

$$-5\partial(B - A)\chi + \frac{1}{4}H\gamma^7\chi = e^{\phi+B}[F\gamma^7 - 2G]\chi . \tag{5.11}$$

Note, all functions are fixed and this equation gives only a constraint. Actually, inserting the expression (5.8) one gets two equations for the two parameter f_0 and f_1 and we found

$$f_0^2 = \frac{1}{8} \quad , \quad f_1^2 = \frac{3}{8} .$$

Note, the mass parameter m drops out on both side and is still a free parameter.

We assumed here that the 2-form F has one selfdual and one anti-selfdual component only. In general one would write $F = e^{2B}[f_i \omega^i + \tilde{f}_j \tilde{\omega}^j]$, where ω^i are the three self-dual 2-forms and $\tilde{\omega}^j$ are the three anti-selfdual 2-forms. The calculation becomes

more involved, but at the end one gets the same equations with the only difference that: $f_0 \rightarrow \pm\sqrt{f_i f_i}$ and $f_1 \rightarrow \pm\sqrt{\tilde{f}_j \tilde{f}_j}$. This result might have been expected, because the two sets of two forms satisfy a quaternionic algebra and can be rotated into one another.

Finally, let us also discuss the equations of motion [since we give the 2-form explicitly, the Bianchi identities are trivially solved]. In the string frame they are given by

$$d[*G] = 0 \quad , \quad d[e^{-2\phi} H] = m F + 12 F \lrcorner G .$$

Note, the rhs of the G -equation is zero because G as well as H have only internal components and since $*G \sim du \wedge dv$, the 4-form equation is trivially solved for our choice of A and B . On the other hand, in order to verify the H -field equation one writes the lhs in components as

$$\frac{1}{\sqrt{g}} \partial_m [\sqrt{g} e^{-2\phi} H^{mpq}] = e^{-4A-6B} \partial_m \left[e^{4A-2\phi} \partial_m e^{2B} \underline{F}_{pq} \right]$$

where \underline{F}_{pq} was the constant 2-form (without the factor e^{2B}) and one uses our equations that imply that B is harmonic and $\partial_m (2A - \phi + B) \partial_m B = \frac{3}{8} m^2 e^{2(B+\phi)}$.

6 Discussion of vacua with SU(3) structures

General SU(3) structures are very restrictive with respect to possible fluxes. For example, for a non-zero mass parameter ($m \neq 0$) the external space can be flat only if there are no fluxes. For $m = 0$, there are two possibilities: either all RR-fields are zero and only the 3-form H -flux is non-zero or only the RR-2-form is nonzero and all other fluxes are trivial. Both cases are known flux vacua that can be obtained from M-theory [13, 19]. Our interest is in finding vacua with SU(3) structures with non-zero mass [23]. In this case one is forced to add a cosmological constant (or superpotential). As we have demonstrated, in this case the internal space has to be nearly Kähler, which are Einstein and compact (for positive scalar curvature). In the following we shall discuss specific explicit examples and their physics implications in more detail.

Fluxes and massive D2-brane

The cone over any nearly Kähler manifold gives a 7-manifold with G_2 holonomy [48], but due to the mass parameter, we cannot identify the 7th direction as the M-theory circle. One should instead identify this additional coordinate as the radial direction of the (external) AdS space. The 10-d metric can thus be interpreted as the geometry of a massive D2-brane where the transversal space has G_2 holonomy with non-zero fluxes. In the string frame, the metric becomes

$$ds_{10}^2 = \frac{1}{\sqrt{H(r)}} \left[-dt^2 + dx_1^2 + dx_2^2 \right] + \sqrt{H(r)} \left[dr^2 + r^2 d\Omega_6^{(NK)} \right]$$

where $d\Omega^{(NK)}$ is the 6-dimensional nearly Kähler metric. Our solution with four unbroken supercharges corresponds to the case where $H \sim (mr)^{-4}$ with constant dilaton so that the space factorizes into $AdS_4 \times \Omega^{(NK)}$. This might be a specific limit of a more general solution, which either preserve only two supercharges, i.e. gives an $\mathcal{N}=1$ vacuum in 3 dimensions, or the complete solution breaks the $SU(3)$ to $SU(2)$ -structure. This is very natural for (non-conical) G_2 -holonomy spaces, because these 7-d spaces admit always for $SU(2)$ structures [49]. Examples of such 2-brane solutions in the presence of fluxes are discussed in [50] or in M-theory in [51], which are however not directly related to our solution, which has a non-zero mass parameter and which has a fixed (constant) dilaton. It would be interesting to explore this direction in more detail, but let us instead discuss some examples of nearly Kähler spaces.

Starting with the corresponding G_2 -holonomy space, one can obtain explicit expressions for the metric and the almost complex structure J of the nearly Kähler 6-manifold; see [52, 53]. There are only a few known coset examples, which are discussed in more detail in [54, 55].

(i) $\frac{G_2}{SU(3)} \simeq S_6$ This is a standard example of a nearly Kähler space, where the cone becomes the flat 7-d space. Note, one can express the 6-sphere also by the coset $SO(7)/SO(6)$ which however breaks supersymmetry.

(ii) $\frac{Sp(2)}{Sp(1) \times U(1)} \simeq \frac{S_7}{U(1)} \simeq \mathbb{CP}_3$ The corresponding G_2 -holonomy space is an \mathbb{R}_3 bundle over S_4 and hence it is the $SO(5)$ invariant metric of \mathbb{CP}_3 appearing here and not the $SU(4)$ -invariant, which is Kähler (instead of nearly Kähler) and hence would break supersymmetry.

(iii) $\frac{SU(3)}{U(1) \times U(1)}$ The cone over this space gives the G_2 -holonomy space related to an \mathbb{R}_3 bundle over \mathbb{CP}_2 and therefore the 6-d metric is $SU(3)$ -invariant. This space is isomorphic to the flag manifold, which again allows for another metric which is Kähler and would break supersymmetry.

(iv) $\frac{SU(2)^3}{SU(2)} \simeq S_3 \times S_3$ In constructing this coset, there are different possibilities of modding out the $SU(2)$ and the nearly Kähler space appearing in our context is obtained by a diagonal embedding yielding as G_2 manifold an \mathbb{R}_4 bundle over S_3 .

Actually there is a whole class of known non-homogeneous examples, which are obtained from G_2 manifold given by an \mathbb{R}_3 bundle over *any* 4-d self-dual Einstein space, where the nearly Kähler space becomes an S_2 bundle over the 4-d Einstein space, which is also known as the twistor space; (i) and (ii) are just the simplest (regular) examples in this class of solutions; other examples can be found in [56, 57]. The appearance of the 4-d base manifold in these G_2 holonomy spaces also explains, why one should expect only $SU(2)$ structures for the complete (massive) D2-brane solution mentioned at the beginning of this section.

Fluxes and intersecting branes

An interesting question is whether this flux vacuum can support a chiral gauge field theory. On the IIA side there are different candidates giving rise to a 4-dimensional field theory: wrapped D4-, D6- or D8-branes. In addition to these D-branes one can also include NS5-branes, which in the massive case have to be endpoints of D6-branes or D4-branes. But before we discuss NS5-branes we will explore the situation where we have added only D-branes in the background of the flux vacuum solution.

Consider first the field theory on (closed) D4-branes, which wrap a 1-cycle inside the nearly Kähler space. The construction of this internal space via G_2 -holonomy spaces excludes a non-contractible S^1 and therefore, D4-branes can only appear in the internal space as dipoles (or higher multipoles) or that the total D4-brane charge vanishes. In fact, due to non-zero RR-flux, the D4-branes will be polarized as known from the Myers effect, where stability is ensured by the non-trivial world volume fields [58]. This implies however, that the world volume spectrum cannot be chiral. The same conclusion holds for D8-branes, that have to wrap a 5-cycle, again contractible

(as the dual 1-cycle), and hence only polarized D8-branes can appear, which however do not give rise to a chiral spectrum.

Wrapped intersecting D6-branes

For D6-branes the situation is different. There are nearly Kähler spaces with a non-trivial third homology class and hence there are 3-cycles upon which one can wrap D6-branes. For the coset examples that we mentioned above, case (iv) has the correct topology for D6-branes to wrap around (different) 3-spheres. As discussed in [51] (see also [55]), this space has three supersymmetric 3-cycles and if one wraps around each of them the same number of D6-branes, they add up to zero in homology, so that the total D6-brane charge is zero. Therefore, due to this geometric property of this nearly Kähler space, there are *no orientifold planes* required to cancel the D6-brane charge. This property is also reflected in cohomology. The D6-brane is charged under the RR-2-form, but since the second cohomology is trivial for this space, the total charge has to vanish. Note, the 2-form $F \sim J$ as well as the 3-form $\text{Re}\Omega$, which “calibrates” the supersymmetric 3-cycle, are not closed, which is a crucial property of the nearly Kähler spaces. These generalized calibrations have been discussed in more detail in [59] and the fact that the calibrating form is not closed means that the volume is not minimized, but the action is extremized [60] [note, the RR-flux background induces on the worldvolume a non-trivial interaction]. The fact that no orientifold projections are required also implies that the geometry is not deformed if one wraps the same number of D6-branes around each 3-cycle – it will only shift the mass parameter. It was moreover shown in [51] that the resulting spectrum on the common intersection of all three stacks of branes wrapping the supersymmetric 3-cycle is chiral. In fact, viewed from the tangent space, the stacks of D6-branes intersect exactly in 120° , giving rise to chiral fermions [39] at the intersection.

If we wrap k D6-branes around each cycle, the resulting gauge group would be $SU(k)^3$. To obtain other gauge groups one can consider orbifolds of the original internal manifold, i.e. the nearly Kähler space (iv) is replaced by $S^3/\mathbb{Z}_m \otimes S^3/\mathbb{Z}_n$, i.e. one wraps the D6-branes not around S^3 but the corresponding Lens space. Note, these orbifolds do not change the local metric and will still solve the equations, but the global structure is different and as a consequence also the gauge group changes.

In order to investigate this in more detail, we have to analyze the wrapping of each stack of D6-branes⁷. One possibility for the three supersymmetric 3-cycles is given by [51]: $\{(1, 0), (0, 1), (-1, -1)\}$ where the two integers parameterize the third homology of the space $S_3 \otimes S_3$. In the case of the orbifold there are now two effects. If the brane wraps the Lens space, say S_3/\mathbb{Z}_m , and the orbifold acts freely, the rank of the gauge group is reduced by a factor of n . On the other hand if the D6-branes are fixed by the \mathbb{Z}_n orbifold, one has to introduce additional Chan-Paton factors and the group is split, ie. the gauge group becomes $U(k)^n$ if we wrap kmn D6-branes on S_3/\mathbb{Z}_m and the transversal space is S_3/\mathbb{Z}_n . The same happens for the other 6-branes wrapping the remaining supersymmetric 3-cycles. Therefore, if we wrap $(k \cdot m \cdot n)$ D6-branes on each of the supersymmetric 3-cycles of $S_3/\mathbb{Z}_m \otimes S_3/\mathbb{Z}_n$, the gauge group will be: $U(k)^n \times U(k)^m \times U(k)^q$, where $q = mn/p$ and $p = \text{gycd}(mn)$. Note that the chiral matter, appearing at the intersection of the two Lens spaces is in the bi-fundamental representation of the corresponding gauge group factors.

Wrapped NS5-branes

A chiral spectrum can also be obtained by allowing for NS5-branes sources, i.e. $dH = n_5 \delta_5$ where n_5 is the number of 5-branes. As a consequence of the Bianchi identities of *massive* supergravity, NS5-branes appear at the boundary of open D6-branes and D4-branes. For a non-compact internal space without fluxes, this setup has been discussed by [61]; let us explore its relevance for our flux vacuum. The chiral spectrum comes here from open strings stretching between the D4- and D6-branes, where the 4-d field theory of interest lives on the boundary of D4-branes and not on D6-branes. Therefore the number of D4-branes is identified with the number of colors N_c and the number of D6-branes (n_6) gives the flavor number. Recall, it is important to have a non-zero mass parameter m which relates the number of branes to each other: $n_6 = m n_5$. To have only one chirality in the open string spectrum, it is important that the D6-branes end on the NS5-branes. Otherwise, ie. if the D6-branes are closed, open strings with both orientations would appear [61]. In string theory, the mass parameter corresponds to D8-branes and this relation can be understood due to creation of D6-branes when the D8-brane passes through the NS5-brane. So the D6-brane ends also on a D8-brane.

⁷We are grateful to Angel Uranga and Ralph Blumenhagen for a discussion on this issue.

In contrast to the case discussed before, we need now non-contractible supersymmetric 2-cycles in order to wrap NS5-branes, i.e. the second Betti number should be non-zero $b_2 \neq 0$. Having these NS5-branes and the mass parameter the D6- and D4-branes are open with the endpoints fixed by the NS5-branes. Therefore, we should not consider the example (iv) from our list of nearly Kähler space, but instead consider the case (ii) or (iii). The example of \mathbb{CP}_3 might be too trivial, since it has only one supersymmetric 2-cycle ($b_2 = 1$) to wrap the NS5-brane. More interesting might be the flag manifold: $\frac{SU(3)}{U(1) \times U(1)}$ which has $b_2 = 2$ and, as in [61], we can wrap two NS5-branes between which D4-branes are stretched and which fix the endpoints of D6-branes. Thus the D4- and D6-branes are stable even without any proper supersymmetric cycle in the internal space. In contrast to the case before, the resulting gauge group is fixed only by the number of D4-branes, which connect the two stacks of NS5-branes; the D6-branes also ending on at NS5-branes give two flavor group factors. Of course, one can also setup more complicated configurations by using spaces with $b_2 > 2$, e.g. by building twistor spaces over the self-dual Einstein space found in [62].

A subtle question is the mass parameter, which is a crucial ingredient of this vacuum solution. If one keeps m just as a parameter and does not introduce D8-branes, the open D6-branes have to end on both sides on the NS5-branes. This however does not yield a chiral spectrum given by 4-6 strings, because for this it is important that the open D6-branes terminate on the NS5-brane only from one side [61]. A better setup is given, if there are polarized or dipole D8-branes, so that the total D8-brane charge is zero [recall, due to the absence of supersymmetric 5-cycle we cannot simply wrap D8-brane on this space] and the D6-branes can stretch between the NS5- and D8-branes.

Moduli fixing

A phenomenological viable model requires that all moduli are fixed. This is especially important in a cosmological setting, because massless scalars can absorb a non-zero vacuum energy. One has to distinguish between closed and open string moduli, where the latter ones are related to the brane position, i.e. they are fixed if the brane wraps a rigid cycle. The closed string moduli on the other hand correspond to deformations

of the geometry. This discussion is very difficult, because the corresponding moduli spaces are not yet well understood in general. But for the simple cases that we discussed in more detail, the moduli space is finite and a discussion is given in [59].

We expect that the closed string moduli are fixed by the (quantized) fluxes that cover all non-trivial cycles of the geometry. Note, due to the back-reaction of fluxes, the geometry of the internal space had to change: from the original flat or Calabi Yau (with no fluxes) to the nearly Kähler one. In the moduli space, this change in geometry is reflected in a lifting of moduli.

But the moduli space is not completely lifted and the remaining moduli should be related to open string moduli. E.g. the $S^3 \times S^3$ geometry has a 3-dimensional moduli space [59], which, if we wrap branes on the supersymmetric 3-cycles, is related to moving around these branes relative to each other. So, although the three supersymmetric 3-cycles are (generalized) calibrated, their relative position to each other is not fixed. We expect something analogous also for the second case, where NS5-branes wrap supersymmetric 2-cycles. The appearance of moduli on the world-volume of the NS5-brane triggers in addition the non-rigidity of the open D4-brane that ends on the two NS5-branes, as well as non-rigid open D6-branes that on one end attach to the NS5-branes, and on the other end attach to the D8-branes which are also not rigid.

Relaxation of the cosmological constant

A negative four-dimensional cosmological constant is a generic feature of supersymmetric flux vacua. In fact, for the massive Type IIA string theory, the internal space with $SU(3)$ structures and a non-zero mass parameter impose that the four-dimensional space to be anti-deSitter. In the 10-d Einstein frame, the negative cosmological constant was given by

$$\Lambda = -e^{4\phi} m^2 = -\left(\frac{H_0}{8G_0}\right)^4 m^2$$

where we inserted the solution for the fixed dilaton in terms of the ratio of the quantized fluxes. Recall, the fluxes H_0 and G_0 were the $SU(3)$ singlet components of the 3-form and 4-form flux introduced in (4.8); they should be quantized if we take into account the conditions of charge quantization of the corresponding branes. By

choosing flux vacua where the appropriate fluxes satisfy ($H_0 \ll G_0$), one can lower the quantized cosmological constant, which however always remains negative.

The goal, however, is to obtain a small positive cosmological constant. Within our framework we shall comment here on two possible scenarios: (*i*) by using the Brown-Teitelboim mechanism of neutralization by membrane instantons [63] or (*ii*) to consider a similar setup as KKLT scenario on the Type IIB side [28]. The crucial ingredient in the latter approach are anti-branes, which increase the vacuum energy and are meta-stable if there are no appropriate fluxes. In the KKLT setup one has anti-D3-branes plus 3-form fluxes and the anti-D3-branes can only decay if they blow up into anti D5/NS5-branes (due to the Myers effect) which decay with the 3-form flux (bubble decay of the vacuum). A small positive cosmological constant can then be obtained by fine tuning. Note, that it is important in this setup that there are no fluxes under which the (anti) branes are charged; otherwise they can immediately decay, i.e. a metastable minimum would not occur. This is exactly what we expect if we apply this scenario to our flux vacuum, where all RR- and NS-NS-fluxes are non-zero. Thus, whether we wrap anti-D6- or anti-NS5-branes, we will not obtain a meta-stable deSitter minimum.

On the other hand, the Brown-Teitelboim mechanism (*i*) is much more appropriate for our supersymmetric vacuum solution. This setup was introduced in string theory in [64, 65, 66] and let us summarize some basic features. The crucial ingredients here are membrane instantons, which relax the cosmological constant in the external space. This is analogous to the decay of an electric field by Schwinger pair creation of charged particles. In Type IIA supergravity we have D2-branes, which appear in the 4-d external space as domain walls and separates regions with a different cosmological constant, where the jump in the cosmological constant is proportional to the brane charge. One does not need to consider strictly D2-branes, also a wrapped brane, which extends in two spatial directions in the external space appears as domain wall across which the cosmological constant jumps. As shown in the above literature in more detail, these instantons can lift the negative cosmological constant to a positive one and subsequently reach a deSitter vacuum. Due to false vacuum decay, this vacuum will not be stable, the most likely configuration is the one with the smallest cosmological constant [66]. In order to argue for a very small positive value, one

needs an additional input. First, the decay from a positive to a negative cosmological constant is gravitationally suppressed [67], but in order to come sufficiently close to a vanishing cosmological constant, one needs a small spacing in the jumps. Bousso and Polchinski [65] argued that a dense discretuum is generated due to multiple flux possibilities appearing in compactification of string or M-theory, but this is not enough to explain the observed smallness. Instead, due to a (weak) anthropic selection they end up with a vacuum of small positive cosmological constant. On the other hand, the implementation of the Brown-Teitelboim mechanism in [66], relies especially on two additional inputs: the brane may wrap an internal degenerate cycle and the world volume dynamics give rise to large density of state factor. Both effects yield a dynamical relaxation of the cosmological constant to even smaller values.

It is straightforward to apply these arguments to our case. One way is to consider D2-branes to relax the cosmological constant and if the NS-flux is sufficiently small, also the dilaton becomes small ($e^\phi \ll 1$) yielding a small spacing in the discrete jumps. But one can also consider other branes, e.g. the D8-branes in our second example (with wrapped NS5-branes), can also appear as domain walls in the external space if they wrap the whole internal space and as in [66] this might yield an efficient relaxation of the cosmological constant. Note however, both scenarios are non-BPS, because the 4-d external space is parallel to a D4-brane and a D2-brane as domain wall is non-BPS configuration. The same holds also for D8-branes, which are non-BPS if they are non-parallel to D4-branes (also D6-branes would not be BPS if they appear as domain walls in the external space). So, whatever brane relaxes the 4-d cosmological constant, it will break supersymmetry.

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References

- [1] A. Strominger, “Superstrings with torsion,” *Nucl. Phys.* **B274** (1986) 253.
- [2] J. Polchinski and A. Strominger, “New vacua for type II string theory,” *Phys. Lett.* **B388** (1996) 736–742, [hep-th/9510227](#).
- [3] S. Gukov, C. Vafa, and E. Witten, “CFT’s from Calabi-Yau four-folds,” *Nucl. Phys.* **B584** (2000) 69–108, [hep-th/9906070](#).
- [4] K. Dasgupta, G. Rajesh, and S. Sethi, “M theory, orientifolds and G-flux,” *JHEP* **08** (1999) 023, [hep-th/9908088](#).
- [5] T. R. Taylor and C. Vafa, “RR flux on Calabi-Yau and partial supersymmetry breaking,” *Phys. Lett.* **B474** (2000) 130–137, [hep-th/9912152](#).
- [6] K. Behrndt and S. Gukov, “Domain walls and superpotentials from M theory on Calabi-Yau three-folds,” *Nucl. Phys.* **B580** (2000) 225–242, [hep-th/0001082](#).
- [7] M. Grana and J. Polchinski, “Supersymmetric three-form flux perturbations on AdS(5),” *Phys. Rev.* **D63** (2001) 026001, [hep-th/0009211](#).
- [8] G. Curio, A. Klemm, D. Lüst, and S. Theisen, “On the vacuum structure of type II string compactifications on Calabi-Yau spaces with H-fluxes,” *Nucl. Phys.* **B609** (2001) 3–45, [hep-th/0012213](#).
- [9] J. Louis and A. Micu, “Type II theories compactified on Calabi-Yau threefolds in the presence of background fluxes,” *Nucl. Phys.* **B635** (2002) 395–431, [hep-th/0202168](#).
- [10] S. Chiossi and S. Salamon, “The intrinsic torsion of $SU(3)$ and G_2 structures,” [math.dg/0202282](#).

- [11] M. Cvetič, G. W. Gibbons, H. Lü, and C. N. Pope, “Almost special holonomy in type IIA and M theory,” *Nucl. Phys.* **B638** (2002) 186–206, [hep-th/0203060](#).
- [12] J. P. Gauntlett, D. Martelli, S. Pakis, and D. Waldram, “G-structures and wrapped NS5-branes,” [hep-th/0205050](#).
- [13] P. Kaste, R. Minasian, M. Petrini, and A. Tomasiello, “Kaluza-Klein bundles and manifolds of exceptional holonomy,” *JHEP* **09** (2002) 033, [hep-th/0206213](#).
- [14] S. Gurrieri, J. Louis, A. Micu, and D. Waldram, “Mirror symmetry in generalized Calabi-Yau compactifications,” *Nucl. Phys.* **B654** (2003) 61–113, [hep-th/0211102](#).
- [15] G. L. Cardoso *et al.*, “Non-Kaehler string backgrounds and their five torsion classes,” *Nucl. Phys.* **B652** (2003) 5–34, [hep-th/0211118](#).
- [16] J. P. Gauntlett, D. Martelli, and D. Waldram, “Superstrings with intrinsic torsion,” [hep-th/0302158](#).
- [17] P. Kaste, R. Minasian, M. Petrini, and A. Tomasiello, “Nontrivial RR two-form field strength and SU(3)-structure,” [hep-th/0301063](#).
- [18] K. Behrndt and M. Cvetič, “Supersymmetric intersecting D6-branes and fluxes in massive type IIA string theory,” *Nucl. Phys.* **B676** (2004) 149–171, [hep-th/0308045](#).
- [19] G. Dall’Agata and N. Prezas, “ $N = 1$ geometries for M-theory and type IIA strings with fluxes,” [hep-th/0311146](#).
- [20] K. Behrndt and C. Jeschek, “Fluxes in M-theory on 7-manifolds and G structures,” *JHEP* **04** (2003) 002, [hep-th/0302047](#).
- [21] K. Behrndt and C. Jeschek, “Fluxes in M-theory on 7-manifolds: G-structures and superpotential,” [hep-th/0311119](#).
- [22] A. R. Frey and M. Grana, “Type IIB solutions with interpolating supersymmetries,” *Phys. Rev.* **D68** (2003) 106002, [hep-th/0307142](#).

- [23] K. Behrndt and M. Cvetič, “General $N = 1$ supersymmetric flux vacua of (massive) type IIA string theory,” [hep-th/0403049](#).
- [24] G. Dall’Agata, “On supersymmetric solutions of type IIB supergravity with general fluxes,” [hep-th/0403220](#).
- [25] M. Becker, K. Dasgupta, A. Knauf and R. Tatar, “Geometric transitions, flops and non-Kaehler manifolds. I,” [hep-th/0403288](#).
- [26] A. R. Frey, “Notes on $SU(3)$ structures in type IIB supergravity,” *JHEP* **06** (2004) 027, [hep-th/0404107](#).
- [27] M. Grana, R. Minasian, M. Petrini, and A. Tomasiello, “Supersymmetric backgrounds from generalized Calabi-Yau manifolds,” [hep-th/0406137](#).
- [28] S. Kachru, R. Kallosh, A. Linde, and S. P. Trivedi, “De Sitter vacua in string theory,” *Phys. Rev.* **D68** (2003) 046005, [hep-th/0301240](#).
- [29] M. Grana, T. W. Grimm, H. Jockers, and J. Louis, “Soft supersymmetry breaking in Calabi-Yau orientifolds with D-branes and fluxes,” *Nucl. Phys.* **B690** (2004) 21–61, [hep-th/0312232](#).
- [30] P. G. Camara, L. E. Ibáñez, and A. M. Uranga, “Flux-induced SUSY-breaking soft terms,” *Nucl. Phys.* **B689** (2004) 195–242, [hep-th/0311241](#).
- [31] D. Lüst, S. Reffert, and S. Stieberger, “Flux-induced soft supersymmetry breaking in chiral type IIB orientifolds with D3/D7-branes,” [hep-th/0406092](#).
- [32] R. Blumenhagen, L. Goerlich, B. Kors, and D. Lust, “Noncommutative compactifications of type I strings on tori with magnetic background flux,” *JHEP* **10** (2000) 006, [hep-th/0007024](#).
- [33] C. Angelantonj, I. Antoniadis, E. Dudas, and A. Sagnotti, “Type-I strings on magnetised orbifolds and brane transmutation,” *Phys. Lett.* **B489** (2000) 223–232, [hep-th/0007090](#).
- [34] G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadan, and A. M. Uranga, “ $D = 4$ chiral string compactifications from intersecting branes,” *J. Math. Phys.* **42** (2001) 3103–3126, [hep-th/0011073](#).

- [35] G. Aldazabal, S. Franco, L. E. Ibáñez, R. Rabadan, and A. M. Uranga, “Intersecting brane worlds,” *JHEP* **02** (2001) 047, [hep-ph/0011132](#).
- [36] R. Blumenhagen, B. Körs, and D. Lüst, “Type I strings with F- and B-flux,” *JHEP* **02** (2001) 030, [hep-th/0012156](#).
- [37] M. Cvetič, G. Shiu, and A. M. Uranga, “Three-family supersymmetric standard like models from intersecting brane worlds,” *Phys. Rev. Lett.* **87** (2001) 201801, [hep-th/0107143](#).
- [38] M. Cvetič, G. Shiu, and A. M. Uranga, “Chiral four-dimensional $N = 1$ supersymmetric type IIA orientifolds from intersecting D6-branes,” *Nucl. Phys.* **B615** (2001) 3–32, [hep-th/0107166](#).
- [39] M. Berkooz, M. R. Douglas, and R. G. Leigh, “Branes intersecting at angles,” *Nucl. Phys.* **B480** (1996) 265–278, [hep-th/9606139](#).
- [40] C. Bachas, “A way to break supersymmetry,” [hep-th/9503030](#).
- [41] S. Kachru, M. B. Schulz, and S. Trivedi, “Moduli stabilization from fluxes in a simple IIB orientifold,” [hep-th/0201028](#).
- [42] S. Kachru, M. B. Schulz, P. K. Tripathy, and S. P. Trivedi, “New supersymmetric string compactifications,” *JHEP* **03** (2003) 061, [hep-th/0211182](#).
- [43] J. F. G. Cascales and A. M. Uranga, “Chiral 4d $N = 1$ string vacua with D-branes and NSNS and RR fluxes,” *JHEP* **05** (2003) 011, [hep-th/0303024](#).
- [44] R. Blumenhagen, D. Lüst, and T. R. Taylor, “Moduli stabilization in chiral type IIB orientifold models with fluxes,” [hep-th/0303016](#).
- [45] L. J. Romans, “Massive $N=2a$ supergravity in ten-dimensions,” *Phys. Lett.* **B169** (1986) 374.
- [46] E. Bergshoeff, R. Kallosh, T. Ortin, D. Roest, and A. Van Proeyen, “New formulations of $D = 10$ supersymmetry and D8 - O8 domain walls,” *Class. Quant. Grav.* **18** (2001) 3359–3382, [hep-th/0103233](#).

- [47] A. Grey and L. Hervella, “The sixteen classes of almost hermitian manifolds and their linear invariants,” *Ann. Math. Pura Appl.* **123** (1980) 35–58.
- [48] C. Bär, “Real Killing spinor and holonomy,” *Comm. Math. Phys.* **154** (1993) 509.
- [49] T. Friedrich, I. Kath, A. Moroianu, and U. Semmelmann, “On nearly parallel $G(2)$ structures,” . SFB-288-162.
- [50] M. Cvetič, H. Lü, and C. N. Pope, “Brane resolution through transgression,” *Nucl. Phys.* **B600** (2001) 103–132, [hep-th/0011023](#).
- [51] B. S. Acharya, F. Denef, C. Hofman, and N. Lambert, “Freund-Rubin revisited,” [hep-th/0308046](#).
- [52] R. Bryant and S. Salamon, “On the construction of some complete metrics with exceptional holonomy,” *Duke Math. J.* **58** (1989) 829.
- [53] G. W. Gibbons, D. N. Page, and C. N. Pope, “Einstein metrics on S^3 , R^3 and R^4 bundles,” *Commun. Math. Phys.* **127** (1990) 529.
- [54] A. Gray, “Nearly Kähler manifolds,” *J. Diff. Geom.* **4** (1970) 283.
- [55] M. Atiyah and E. Witten, “M-theory dynamics on a manifold of $G(2)$ holonomy,” *Adv. Theor. Math. Phys.* **6** (2003) 1–106, [hep-th/0107177](#).
- [56] M. Cvetič, G. W. Gibbons, H. Lu, and C. N. Pope, “Bianchi IX self-dual Einstein metrics and singular $G(2)$ manifolds,” *Class. Quant. Grav.* **20** (2003) 4239–4268, [hep-th/0206151](#).
- [57] K. Behrndt, G. Dall’Agata, D. Lust, and S. Mahapatra, “Intersecting 6-branes from new 7-manifolds with $G(2)$ holonomy,” *JHEP* **08** (2002) 027, [hep-th/0207117](#).
- [58] R. C. Myers, “Dielectric-branes,” *JHEP* **12** (1999) 022, [hep-th/9910053](#).
- [59] J. Gutowski, S. Ivanov, and G. Papadopoulos, “Deformations of generalized calibrations and compact non-Kähler manifolds with vanishing first Chern class,” [math.dg/0205012](#).

- [60] J. F. G. Cascales and A. M. Uranga, “Branes on generalized calibrated submanifolds,” `hep-th/0407132`.
- [61] A. Hanany and A. Zaffaroni, “Chiral symmetry from type IIA branes,” *Nucl. Phys.* **B509** (1998) 145–168, `hep-th/9706047`.
- [62] D. M. J. Calderbank and H. Pedersen, “Selfdual Einstein metrics with torus symmetry,” `math.dg/0105263`.
- [63] J. D. Brown and C. Teitelboim, “Neutralization of the cosmological constant by membrane creation,” *Nucl. Phys.* **B297** (1988) 787–836.
- [64] R. Bousso and A. Chamblin, “Open inflation from non-singular instantons: Wrapping the universe with a membrane,” *Phys. Rev.* **D59** (1999) 063504, `hep-th/9805167`.
- [65] R. Bousso and J. Polchinski, “Quantization of four-form fluxes and dynamical neutralization of the cosmological constant,” *JHEP* **06** (2000) 006, `hep-th/0004134`.
- [66] J. L. Feng, J. March-Russell, S. Sethi, and F. Wilczek, “Saltatory relaxation of the cosmological constant,” *Nucl. Phys.* **B602** (2001) 307–328, `hep-th/0005276`.
- [67] S. R. Coleman and F. De Luccia, “Gravitational effects on and of vacuum decay,” *Phys. Rev.* **D21** (1980) 3305.